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Passive vibration control: a structure–immittance approach

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Linear passive vibration absorbers, such as tuned mass dampers, often contain springs, dampers and masses, although recently there has been a growing trend to employ or supplement the mass elements with inerters. When considering possible configurations with these elements broadly, two approaches are normally used: one structure-based and one immittance-based. Both approaches have their advantages and disadvantages. In this paper, a new approach is proposed: the structure–immittance approach. Using this approach, a full set of possible series–parallel networks with predetermined numbers of each element type can be represented by structural immittances, obtained via a proposed general formulation process. Using the structural immittances, both the ability to investigate a class of absorber possibilities together (advantage of the immittance-based approach), and the ability to control the complexity, topology and element values in resulting absorber configurations (advantages of the structure-based approach) are provided at the same time. The advantages of the proposed approach are demonstrated through two case studies on building vibration suppression and automotive suspension design, respectively.

1. Introduction

Traditionally passive dynamic absorbers have made use of mechanical elements including masses, dampers and

springs. An example is the tuned mass damper (TMD) proposed by Frahm [1], which has been installed or proposed for damping excessive vibration in a variety of engineering problems [2–5]. Recently, in 2002, a new passive mechanical element termed the inerter has been introduced by Smith [6]. It has the property that the generated force is proportional to the relative acceleration across its two terminals. This contrasts with the mass element which, by definition, always has one terminal connected to the ground. The inerter allows electrical networks (controllers) to be translated over to mechanical ones in a completely analogous way. For passive electrical networks, there is a well-known theorem in the field of network synthesis proposed by Bott & Duffin [7] showing that any *positive-real* functions could be realized as the driving-point immittance of a network consisting of only resistors, capacitors and inductors. Hence, such positive-real functions can also be synthesized by mechanical networks consisting of dampers, inerters and springs. Beneficial inerter-based dynamic absorbers have been identified for a wide range of mechanical structures, such as automobiles [8], railway vehicles [9–11] and buildings [12–16]. Apart from pure mechanical absorbers, mechatronic designs, which enable the controller's immittance to be realized through a combination of mechanical and electrical networks, have also been proposed and proved to be beneficial [17–19]. In general, there are two approaches to identify beneficial passive vibration controllers; the *structure-based* and the *immittance-based* approaches. Each has its own advantages and limitations.

In the structure-based approach, network layouts, which represent the topological arrangement of components, are first proposed. Parameter values for each of the elements are then selected using performance criteria while taking into account any component parameter constraints. The advantage of this approach lies in the predetermined complexity (number of elements) and topology (how the elements are connected with each other) of candidate layouts. Hence, normally the obtained absorbers are simple layouts, and relatively easier to manufacture. For example, the tuned inerter damper (TID) [12], tuned viscous mass damper (TVMD) [13] and tuned mass-damper-inerter (TMDI) [14] have been identified as building suspensions. However, despite the fact that there are countless possible network layouts, only one layout can be covered by the structure-based approach. This inevitably limits the achievable performance of mechanical systems. The immittance-based approach is far more systematic.

With the immittance-based approach, one can obtain a positive-real immittance function which is capable of providing the optimum performance. Then, the corresponding network layout and element values are identified using network synthesis theory. In this way, a wide range of candidate layouts are covered. In previous studies, by assuming positive-real biquadratic immittance functions as controllers, and using relevant network synthesis theory (e.g. [20,21]), configurations which can provide significant performance improvements for railway vehicles [22] and landing gears [23] have been identified. Unfortunately, the network complexity and topology cannot be pre-restricted by the immittance-based approach. Hence undesirable networks, such as ones with excessive numbers of components or component parameter values, can be obtained. As an example, some positive-real biquadratic immittance require the Bott–Duffin synthesis [7], which corresponds to the series–parallel networks with nine elements. Furthermore, mechanical and electrical absorber realizations have respective difficulties. For example, dampers and inerters with large parameter values are less preferable; similarly, large inductance values are difficult to manufacture. Unlike the structure-based approach, it is not possible to fix or constrain any element value in the resulting network using the immittance-based approach.

This paper proposes a new design method, the *structure–immittance* approach, for passive vibration control. With this approach, a new class of immittance functions, the structural immittances is obtained. These functions cover a full set of network layouts with a predetermined number of each element type, and contain explicit information for each topology possibility. At the same time, the element value can be fixed or constrained. In this way, advantages of both the structure-based and immittance-based approaches are preserved. The proposed method is applicable to both mechanical and electrical absorber layouts.

This paper is structured as follows. In §2, a method for formulating the structural immittances, capable of realizing all possible networks containing a predetermined number of elements, is

proposed. In addition to describing the approach, two examples are provided to illustrate it. Applications of the structure–immittance approach to a civil engineering and an automotive control example are shown in §3. These are used to illustrate the advantages of the new technique over both the structure-based and the immittance-based approaches. Conclusions are drawn in §4.

2. Candidate layouts formulation

In this paper, we consider the one-port (two-terminal) series–parallel networks in mechanical systems. This section defines the structural immittances, that capture all possible arrangements that consist of a fixed number of each element type. Development of the method will be in terms of mechanical networks. A two-terminal mechanical network is shown in figure 1, where an equal and opposite force F is applied and results in a relative velocity $v = v_2 - v_1$ (or vice versa). Using the force–current (or mobility) analogy [24–26], the proposed method can also be applied directly in the electric domain. This analogy is chosen to preserve the topology of the elements in the network. Note that the consequence of preserving topology is that the resulting transfer function for the mechanical networks $Y(s) = \tilde{F}(s)/\tilde{v}(s)$,¹ where $\tilde{F}(s)$ and $\tilde{v}(s)$ are the force at the terminals and the relative velocity across the terminals in the Laplace domain, maps to the electrical admittance, $Y(s) = \tilde{I}(s)/\tilde{U}(s)$, where $\tilde{I}(s)$ and $\tilde{U}(s)$ are the current and voltage, respectively.

(a) Pragmatic example

Consider the possible layouts with one damper and one inerter. Just two networks can be obtained by connecting damper in series or in parallel with the inerter (table 1, appendix A). This can be represented by the *generic network* shown in figure 2 with the condition that either $c_1 = 0$ and $0 < c_2 < \infty$ or $0 < c_1 < \infty$ and $c_2 = \infty$ to cover the two possible layouts. Note that the constructed network is not unique. For example, we could also have a network which has c_1 in parallel with a series connection of b and c_2 or a network with one damper and two inerters.

The transfer function from force to velocity of the constructed network can be expressed as

$$\frac{F}{v} = \frac{c_2(bs + c_1)}{(bs + c_1 + c_2)} \quad (2.1)$$

with the condition that one of the parameters c_1 or $1/c_2$ is positive and the other one equals zero. The two possible networks shown in table 1 can be analysed using (2.1) with the constraint on c_1 and $1/c_2$. While this process is straightforward for the one-damper and one-inerter system, a general formulation is needed as the numbers of elements become larger.

(b) General formulation

For the general formulation of structural immittances, three elements p, q and r are considered. They can represent in any order either inerters, dampers and springs in the mechanical domain or capacitors, inductors and resistors in the electrical one. Assume the numbers of p, q and r elements are P, Q and R , respectively, and $P \leq Q \leq R$ is satisfied by selecting the p, q and r elements appropriately. The formulation of a generic network representing P p , Q q and R r elements requires a series of steps, some of which are iterated. Before the steps are described, we define the terminology used.

In the procedure, to generate the generic network, the most numerous element r is appointed as the *added* element, and the other two elements p, q are chosen as the *base* elements. From figure 2, in which r represents dampers, it can be seen that the generic network has more dampers than is allowed, hence requires the condition that either $c_1 = 0$ or $c_2 = \infty$ to recreate the desired one-damper, one-inerter layouts. We say that an element is *present* if its value is positive and

¹We avoid naming this transfer function as there is an inconsistency in the literature. In some rm s , mechanical impedance is defined as v/F [27,28]; however, others adopt the terminology that mechanical impedance is F/v and its inverse is termed mobility or admittance [29,30].

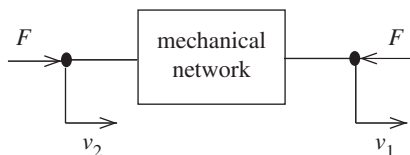


Figure 1. A one-port mechanical network with equal and opposite force F and velocities at the two terminals v_1, v_2 .

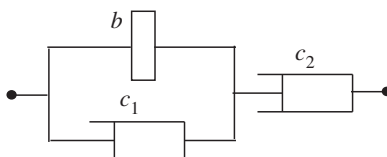


Figure 2. The generic network for the one-damper and one-inerter case.

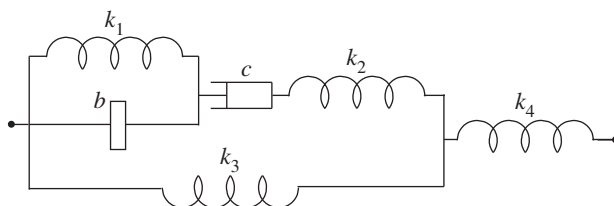


Figure 3. Network example to demonstrate element removal.

finite. An element that is not present is said to be *removed* and takes the value of 0 or ∞ —the choice of value is made to ensure that no other elements are locked rigid (or short-circuited in the electrical domain) and that the terminals are not disconnected. Taking the network of figure 3 as an example, if k_1 is removed, it is set to be zero; otherwise the terminals of inerter will be rigid. For k_2 , we must set $k_2 = \infty$; otherwise the terminals of the damper and k_4 will be disconnected. Using similar argument, the removal of k_3 and k_4 corresponds to $k_3 = 0$ and $k_4 = \infty$, respectively.

In the procedure, we also use the term *generic sub-network* $p_g(q_g)$, as a network consisting of one base element $p(q)$ and $R + 1$ number of r elements, with the condition that at most R number of r elements are present. This sub-network is generated by adding to the base element an r element in parallel, then adding a second in series to the resulting network, and then a third in parallel and so on until $R + 1$ r elements have been added. The need to add $R + 1$ elements for all possible combinations to be captured leads to the condition that at least one r element must be removed in the final network. For example, consider the case with one damper, one inerter and two springs. As the most numerous element is spring, it is appointed as the added element and, accordingly, the other two elements, damper and inerter, are the base elements. The two generic sub-networks are shown in figure 4. For both sub-networks a condition that at most two of the three springs may be present is imposed. Note that starting the adding sequence with an r element in series, then adding an r element in parallel and in series sequentially until $R + 1$ r elements have been added, generates a different diagram; however, applying the condition that only R r elements are present results in the same set of networks.

The general formulation of generic networks representing P p , Q q and R r elements is detailed in the following steps and summarized in figure 5:

- Step 1. Labels p , q and r are assigned based on the number of each element available using $P \leq Q \leq R$. Then, $P + Q$ generic sub-networks will be formulated for the $P + Q$ base elements.

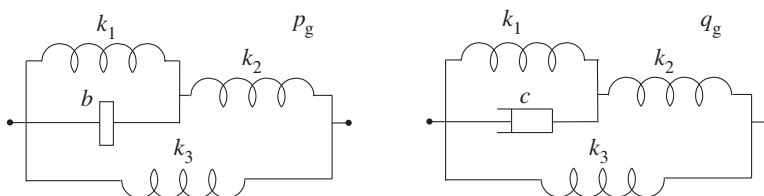


Figure 4. The two generic sub-networks for a single damper (p) and inerter (q) and two springs (r), where for either network at most two of the three springs may be present.

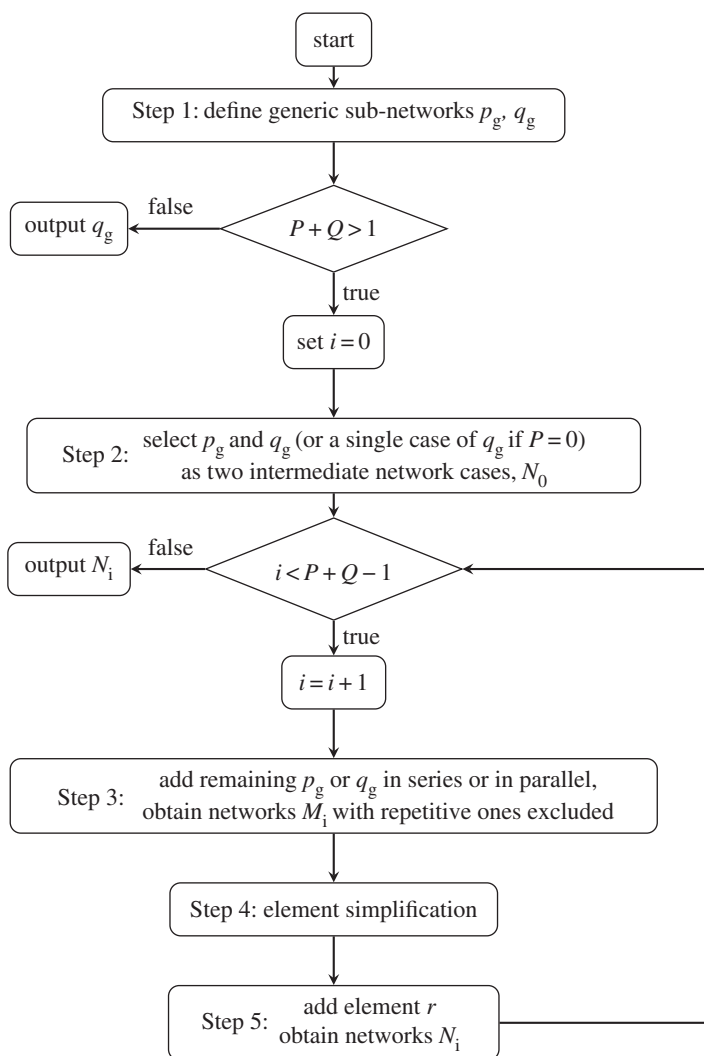


Figure 5. Flowchart summarizing the steps for obtaining generic networks for Pp, Qq and Rr case where $P \leq Q \leq R$.

- Step 2. From the P p_g generic sub-networks and the Q q_g ones, if $P \geq 1$, $Q \geq 1$, one p_g and one q_g are selected as intermediate network cases. If P equals 0, only one q_g is chosen. (If both $P = Q = 0$, then the network is just a single r element).
- Step 3. For each intermediate network case, we first check the number of p and q elements. If the number of p elements is less than P , two new more developed networks are

generated, one with a p_g added in series to the existing intermediate network and the other where it is added in parallel. The same is done for the q elements. This increases the number of intermediate network cases by up to a factor of 4. Any repeated network should be omitted at this step, for example, at $i = 1$ adding a p_g to a q_g in series is the same as adding a q_g to a p_g in series. The new more developed networks are denoted as M_i , and, as always, are subject to the condition that at most R number of r elements are present.

- Step 4. Element simplification is the process of removing any redundant r elements from each intermediate network obtained in Step 3. For an intermediate network, first, two or more r elements connected in series or in parallel with each other are reduced to a single r element. Then, for the resulting network, each r element, r_1, r_2, \dots, r_n (assuming that the number of r elements in the network is n), is checked in turn. Consider element r_1 ; it is redundant if any network obtained with $0 < r_1 < \infty$ and with $R - 1$ of the other elements r_2, \dots, r_n being present can be realized using R of the other elements r_2, \dots, r_n . If r_1 is redundant, it is removed. Then we move on to consider the next element, r_2 , in the modified structure. This process will be ended when all n of the r elements have been checked for redundancy for each of the intermediate networks.
- Step 5. Additional elements r are now added in series and in parallel with the networks obtained from Step 4. Each time, a single element r is added, it is then checked to see if it is redundant or not. We choose to first add an r element in series, although we note that same networks are covered if an element is added in parallel first. The networks N_i with the condition that at most R number of the r elements present are obtained when the addition of any more r elements does not result in any new networks.

There are three ways for the process to end. If $P + Q = 0$, then the generic network is simply an r element (not shown in figure 5). If $P + Q = 1$, i.e. $Q = 1$, then the output is q_g . If $P + Q > 1$, then the output generic networks are the N_i networks, each with P p , Q q elements and the condition that R of the r elements are present. These generic networks are now used to write the structural immittances, which are the transfer functions from force to velocity for each of these networks.

The algorithm in figure 5 is designed to be applicable to networks with P p , Q q and R r elements. There is no restriction in the numbers of P , Q and R . The construction procedure is motivated by a previous work [31], where all series–parallel networks with two reactive elements and an arbitrary number of resistive elements are studied. Both the arguments in the present work and in [31] made use of the fact that any series–parallel network can be constructed by a sequence of steps, where at each step two existing elements or two-terminal networks are connected in series or in parallel to obtain a new network. In [31], it is explained that at some stage the two reactive elements are combined together, whereas, before this step, they belong to different networks. And after this step, any subsequent step involves a series or parallel connection of a damper (resistor) with the network obtained in the previous step. In this paper, the procedure is generalized; the base element P p and Q q are in separate generic sub-networks (Step 1), and then their connection in all possible sequences and arrangements are taken into consideration (Steps 2 and 3). The remaining added elements r are then included (Step 5). Following this procedure, all series–parallel networks that can be formed using P p , Q q and R r elements are covered.

Implementing the algorithm in figure 5 in a computer is of considerable interest, especially for networks with more numbers of elements. In the algorithm, Steps 1–3 should be straightforward to be implemented. The difficulty lies in Steps 4 and 5 when checking the redundancy of the added elements r . At the moment, the redundant r elements are identified by comparing the resulting network layouts. If all layouts introduced by a specific r element present are covered by other layouts considered in previous steps, this r element is removed. We anticipate this would be difficult to code; however, we observe that while simplifying the resulting generic networks, these steps are not necessary. The effect of using generic networks with some degree of redundancy would not affect the optimization results, although there may be more than one minima of cost function that result in identical configurations once the redundancy is noted. If one wished to

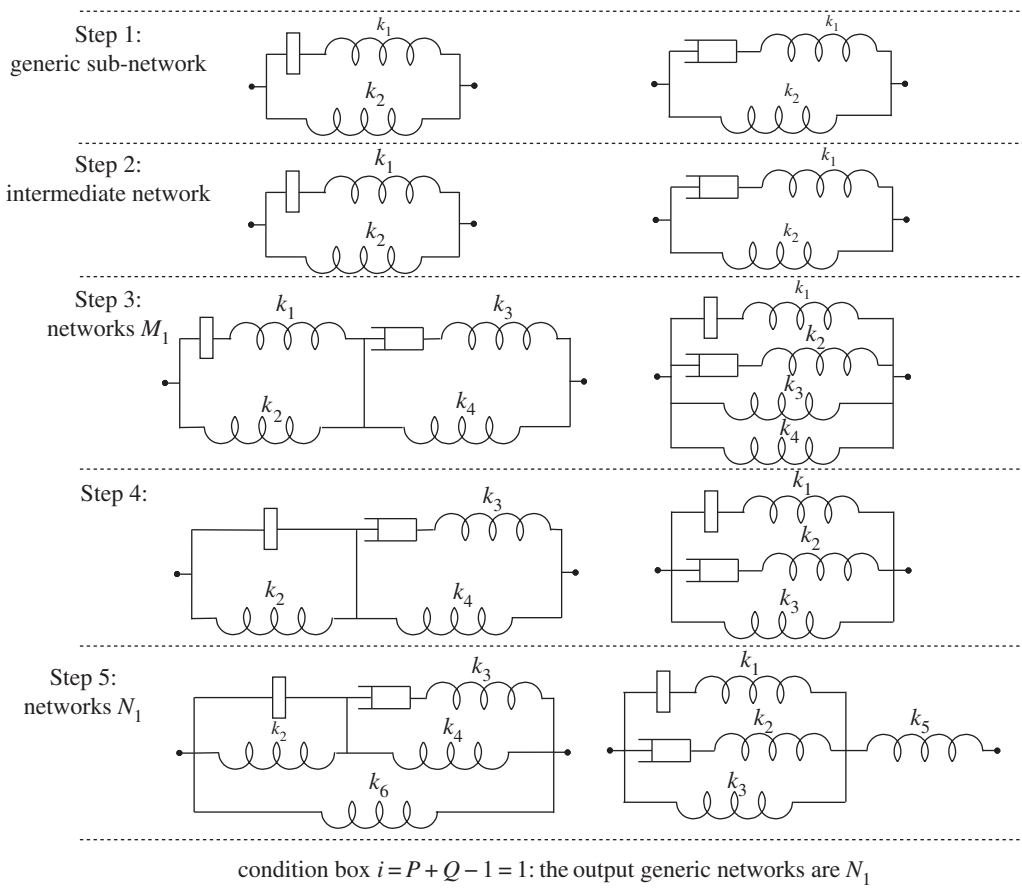


Figure 6. The networks obtained at Steps 1–5 for one-damper, one-inerter and one-spring case.

pursue the algorithmic implementation of the redundancy, the Boolean representation reported in [32] may offer a way forward.

(c) Case demonstration

In this section, two cases, one with one damper, one inerter and one spring and the other with one damper, one inerter and two springs, are analysed, respectively. The structural immittances for each case are formulated based on the process shown in figure 5. The obtained network sets have been double checked using the results in [33].

(i) For the case with one damper, one inerter and one spring, we have $P = Q = R = 1$. We choose the damper and inerter as the base element for the sub-networks and the spring is appointed as the added element. Based on figure 5, the networks obtained at each step are shown in figure 6.

At Step 1, two generic sub-networks, p_g and q_g , are obtained; see figure 6-Step 1. For both of these networks, at most one of $1/k_1$, k_2 is positive and the others equal zero. As $P + Q = 2 > 1$, at Step 2, one p_g and one q_g sub-network are selected as the intermediate network cases, see figure 6-Step 2. Four networks can first be obtained at Step 3, where two are obtained by adding q_g in parallel or in series with p_g and the other two are constructed by connecting p_g in parallel or in series with q_g . Note that adding a p_g (q_g) sub-network to the p_g (q_g) intermediate network is not an option as $P = Q = 1$ here. It can be seen that two of the four networks are repetitive, hence we obtain two networks, denoted as M_1 , shown in figure 6-Step 3. For the left-hand network of figure 6-Step 3, at most one of the parameters $1/k_1$, k_2 , $1/k_3$ and k_4 is positive and the others are

all zero. For the other network of figure 6-Step 3, at most one of $1/k_1$, $1/k_2$, k_3 and k_4 is positive with the others equal to zero.

At Step 4, we need to check the redundancy of the springs in the two networks obtained in Step 3. Considering first the left-hand network of figure 6-step 3, the springs are checked one by one. Assuming $0 < k_1 < \infty$ and $k_2 = 0$, $k_3 = \infty$, $k_4 = 0$, a network with inerter, damper and spring in series connection can be obtained; this can also be realized by the condition $k_1 = \infty$, $k_2 = 0$, $0 < k_3 < \infty$ and $k_4 = 0$. Hence, k_1 is redundant and a modified network structure with $k_1 = \infty$ can be obtained. Moving on to spring k_2 in the modified structure, the network obtained with $0 < k_2 < \infty$, $k_3 = \infty$, $k_4 = 0$ is not equivalent to any network obtained with one of the springs k_3 , k_4 being present, and hence is not redundant. Then considering the k_3 and the k_4 , it can be seen that these two cannot be removed either. As a result, this network can be simplified to the left-hand network of figure 6-Step 4, with the condition that at least two of the parameters k_2 , $1/k_3$ and k_4 are equal to zero. For the right-hand network of figure 6-Step 3, first, k_4 can be removed since it is in parallel with k_3 and then following a similar argument, it can be checked that the springs k_1 , k_2 and k_3 are not redundant. Hence, the right-hand network in figure 6-Step 4 can be obtained.

Step 5 involves connecting springs in series or in parallel with the networks of figure 6-Step 4. Taking the left-hand network from Step 4 as an example, consider adding a spring k_5 in series; it can be checked that the network obtained with $0 < k_5 < \infty$, $k_2 = 0$, $k_3 = \infty$, $k_4 = 0$ is equivalent to that with $k_2 = 0$, $0 < k_3 < \infty$, $k_4 = 0$, $k_5 = \infty$, hence the new spring k_5 is redundant. Instead, consider a spring k_6 added in parallel; this time the network obtained with $0 < k_6 < \infty$ cannot be realized by any other springs. Thus, the parallel spring k_6 is needed. Any additional spring added in series or in parallel is redundant because those can be covered by the presence of either k_3 or k_6 , respectively. As a result, the left-hand network of figure 6-Step 5 can be obtained, which requires at least three of the parameters k_2 , $1/k_3$, k_4 and k_6 to equal zero. Similarly, for the right-hand network of figure 6-Step 4, we can obtain the network shown in the right-hand side of figure 6-Step 5, with the condition that at least three of the parameters $1/k_1$, $1/k_2$, k_3 and $1/k_5$ are all zero.

By checking, $i = 1$ does not satisfy the condition $i < P + Q - 1$, the networks of figure 6-Step 5 covers the full set of networks with one damper, one inerter and one spring (shown in table 2 in the appendix), with the condition that one and only one of the springs is present.

The final step in the proposed structure-immittance process is to derive transfer functions based on these network layouts for optimization. By expressing the force-velocity transfer functions of the two networks shown in figure 6-Step 5 and making use of the condition that at most one spring is present, we can obtain the following:

Between the two force-velocity structural immittances:

$$Y_1(s) = \frac{bcs^2 + b(k_4 + k_6)s + c(k_2 + k_6)}{bc(1/k_3)s^3 + bs^2 + cs + k_4} \quad (2.2)$$

and

$$Y_2(s) = \frac{bc(1/k_1 + 1/k_2)s^3 + bs^2 + cs + k_3}{b(1/k_1 + 1/k_5)s^3 + c(1/k_2 + 1/k_5)s^2 + s}. \quad (2.3)$$

where $b \geq 0$ and $c \geq 0$, the full set of networks with at most one damper, one inerter and one spring, (see appendix A, table 2), can be realized. And since at most one spring of the networks shown in figure 6-Step 5 is present, for the function (2.2), at least three of the parameters k_2 , $1/k_3$, k_4 , k_6 must equal zero, whereas for (2.3), at least three of the parameters $1/k_1$, $1/k_2$, k_3 , $1/k_5$ must equal zero.

(ii) For the case with one damper, one inerter and two springs, we set $P = 1$, $Q = 1$ and $R = 2$ with the damper and inerter as the base elements and the springs as the added element. The networks obtained following the flowchart of figure 5 have been shown in figure 7. Again note that as $P = Q = 1$ in this example, no iteration of the procedure for $i \geq 1$ is needed. Also note that the networks obtained in every step must satisfy the condition that at most two springs are present. A full description of the steps is not given as the procedure is the same as the

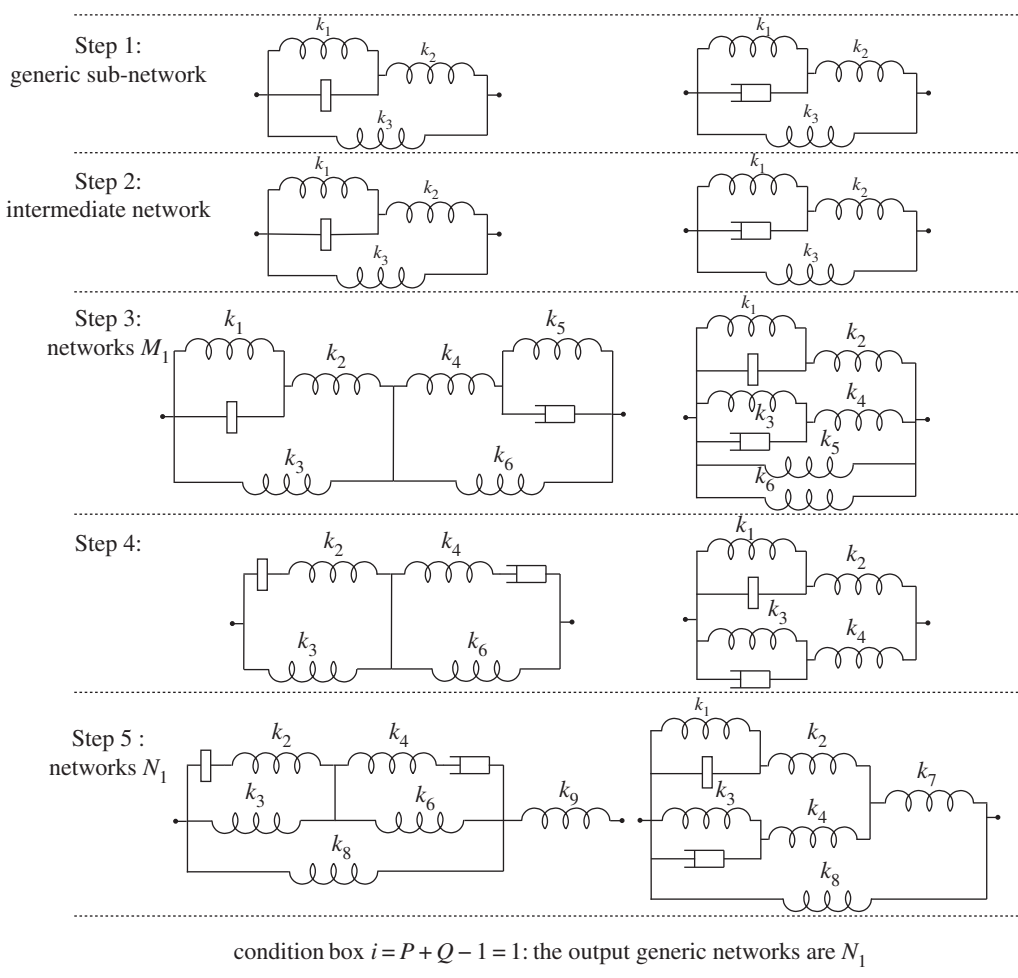


Figure 7. The networks obtained at Steps 1–5 for one-damper, one-inerter and two-springs case.

previous example. However, the element simplification in Step 4 and adding springs in Step 5 is not straightforward for this case, and so is worth describing here.

Considering the left-hand network of figure 7-Step 3, first, for k_1 , it can be seen that the network obtained with k_1 and k_2 present is equivalent to that obtained with k_3 and k_4 present. When k_1 and k_3 are present, they can be reduced to one spring, and the obtained network can be realized by k_3 alone being present. For k_1 and k_4 being present, the network is equivalent to that obtained with k_3 and k_4 being present. It can also be checked that the network with k_1 and k_5 or k_1 and k_6 being present can be realized with k_3 and k_6 being present. As a result, k_1 is redundant and it can be removed by setting $k_1 = 0$. We then move on to consider k_2 in the modified structure. It can be seen that the network obtained with k_2 and k_3 present cannot be realized by any other two springs, so k_2 is not redundant. Similarly, k_3 is needed. Then it can be checked that the network with k_4 and k_6 present is not equivalent to any other networks, so k_4 is not redundant. Then it can be shown that k_5 is redundant and k_6 is needed. Hence, the left-hand network of figure 7-Step 4 can be obtained, which requires at most two of the parameters $1/k_2$, k_3 , $1/k_4$ and k_6 to be positive and the others equal zero. For the right-hand network from Step 3, following a similar simplification process, we can obtain the right-hand network of figure 7-Step 4, with the condition that at most two springs are present.

In Step 5, we first analyse the left-hand network of figure 7-Step 4. By adding a spring k_7 in series, it can be seen that the network with k_2 and k_7 or k_4 and k_7 present is equivalent to that with

k_2 present. It can be further checked that network obtained with k_3 and k_7 or k_6 and k_7 can also be realized by k_3 and k_4 or k_2 and k_6 , respectively. Hence, k_7 can be removed with $k_7 = \infty$. Then with another spring, k_8 , in parallel, new networks can be obtained, e.g. the network with k_2 and k_8 being present cannot be realized with any other pair of springs. Next, adding an additional spring k_9 in series with the new network, we can see that the network obtained with k_8 and k_9 present is a new network, so k_9 is needed. After that, it can be shown any additional spring added in parallel or series is redundant, hence the left-hand network of figure 7-Step 5 can be obtained, with the condition that all but two of the parameters $1/k_2, k_3, 1/k_4, k_6, k_8$ and $1/k_9$ are set to zero. Following a similar argument, the right-hand network of figure 7-Step 5 can be formulated from that of Step 4, with the condition that at most two springs are present. The full set of networks, 18 in total, with one damper, one inerter and two springs is shown in appendix A, table 3, and it can be checked that all of them are included within by the generic networks of figure 7-Step 5. These two generic networks may be represented by force-velocity structural immittances:

$$Y_i(s) = \frac{n_i(s)}{m_i(s)} \quad (i = 1, 2), \quad (2.4)$$

where

$$\begin{aligned} n_1(s) &= bc \left(\frac{k_3}{k_2} + \frac{k_8}{k_2} + \frac{k_6}{k_4} + \frac{k_8}{k_4} + 1 \right) s^3 + b(k_6 + k_8)s^2 \\ &\quad + c(k_3 + k_8)s + k_3k_6 + k_3k_8 + k_6k_8, \\ m_1(s) &= s \left(bc \left(\frac{1}{k_2} + \frac{1}{k_4} + \frac{1}{k_9} \right) s^3 + b \left(\frac{k_3}{k_2} + \frac{k_6}{k_2} + \frac{k_6}{k_9} + \frac{k_8}{k_9} + 1 \right) s^2 \right. \\ &\quad \left. + c \left(\frac{k_3}{k_4} + \frac{k_3}{k_9} + \frac{k_6}{k_4} + \frac{k_8}{k_9} + 1 \right) s + k_3 + k_6 \right) \\ n_2(s) &= bc \left(\frac{1}{k_2} + \frac{1}{k_4} \right) s^3 + b \left(\frac{k_3}{k_2} + \frac{k_3}{k_4} + \frac{k_8}{k_4} + \frac{k_8}{k_7} + 1 \right) s^2 \\ &\quad + c \left(\frac{k_1}{k_2} + \frac{k_8}{k_4} + \frac{k_1}{k_4} + \frac{k_8}{k_7} + 1 \right) s + k_1 + k_3 + k_8, \\ m_2(s) &= s \left(bc \left(\frac{1}{(k_2k_4)} + \frac{1}{(k_2k_7)} + \frac{1}{(k_4k_7)} \right) s^3 + b \left(\frac{1}{k_2} + \frac{1}{k_7} \right) s^2 \right. \\ &\quad \left. + c \left(\frac{1}{k_4} + \frac{1}{k_7} \right) s + \frac{k_1}{k_2} + \frac{k_1}{k_7} + \frac{k_3}{k_4} + \frac{k_3}{k_7} + 1 \right). \end{aligned}$$

Here, $b \geq 0$, $c \geq 0$ and for $i = 1$, corresponding to the left-hand generic network in figure 7, at least four of parameters $1/k_1, k_3, 1/k_4, k_6, k_8, 1/k_9$ equal zero, whereas with $i = 2$, corresponding to the right-hand network in figure 7, at least four of parameters $k_1, 1/k_2, k_3, 1/k_4, 1/k_7, k_8$ equal zero.

These transfer functions can be optimized for certain performance criteria. Importantly, using the method proposed here, this optimization is done in the knowledge that the resulting transfer function can be realized with a layout of specified complexity. In addition, with the explicit link from the transfer function, $Y_i(s)$, to the component values, parameters in the transfer function can be limited as optimization constraints to ensure acceptable element values. As will be seen in the following examples, the optimization of the transfer functions is conducted over a multi-parameter space; however, with the constraints applied, this space is reduced in complexity, allowing all the layouts to be considered at one time. Using the structural immittances as the force to velocity transfer function of the vibration absorber, all the possible layouts with predetermined complexity and restricted range of element values can be analysed systematically, and by optimizing the objective functions considered, the optimal configurations satisfying the predetermined constraints can be obtained. In addition, the sensitivity of the obtained structure to parameter variations can be investigated with the proposed approach, which can be seen from the building application in the next section.

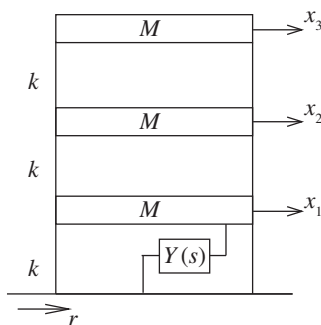


Figure 8. The three storey building model with a vibration suppression device mounted at the bottom.

(d) Inclusion of mass elements

So far networks that consist of inerters, dampers and springs (or via force–current analogy, capacitors, resistors and inductors) have been discussed. However, in mechanical vibration suppression, the mass element has also traditionally been used, for example TMDs. For a vibration control system including physical mass(es), the complexity lies in the fact that the immittance functions considered in this work can only represent one-port mechanical networks which preserve the property of equal and opposite force at the terminals (see figure 1). However, the proposed structure–immittance approach can still be applied by treating the masses as optimizable parameters within the dynamic system to be controlled. For example, a traditional TMD can be characterized using a transfer function $Y(s)$ (corresponding to a spring–parallel–damper layout) connected with a mass. If both ends of an attached mass are connected with suspension elements, then the proposed approach can still be used but now with more than one transfer function. An example involving a mass element in the suppression system is provided at the end of §3a.

3. Application examples

The following two examples illustrate the application of the results obtained in the previous section. The first example is to design a vibration suppression device mounted at the bottom of a simplified three-storey building model. The range of the preferred inertance and damping values have been specified and the optimal absorber configuration with a predetermined number of each elements has been obtained with respect to the inertance and damping values. The second example uses a quarter-car model [8,27] to demonstrate the advantage of the structure–immittance approach. As discussed in the introduction, for the building application, many device layouts have been considered individually [12,13], while using the method presented here, the investigation and comparison can be carried out across all possible layouts with predetermined complexity in terms of number of each element type. Likewise multiple automotive suspension layouts have been identified, which will be discussed in detail in §3b.

(a) Vibration suppression device for multi-storey building

Consider the three-storey building model shown in figure 8, with floor masses M and the inter-storey elasticity of the structure being represented by stiffness k . The structural damping is taken to be zero as it is typically negligible compared with that introduced by the control device. Here, we consider adding a passive mechanical suppression system between the ground and the first floor. The dynamics of the control device are represented by the transfer function $Y(s) = F/v$, where F is the force exerted by the control device and v is the velocity between the two terminals. Here, we adopt the building parameters used in [12,16], namely $M = 1000$ kg and

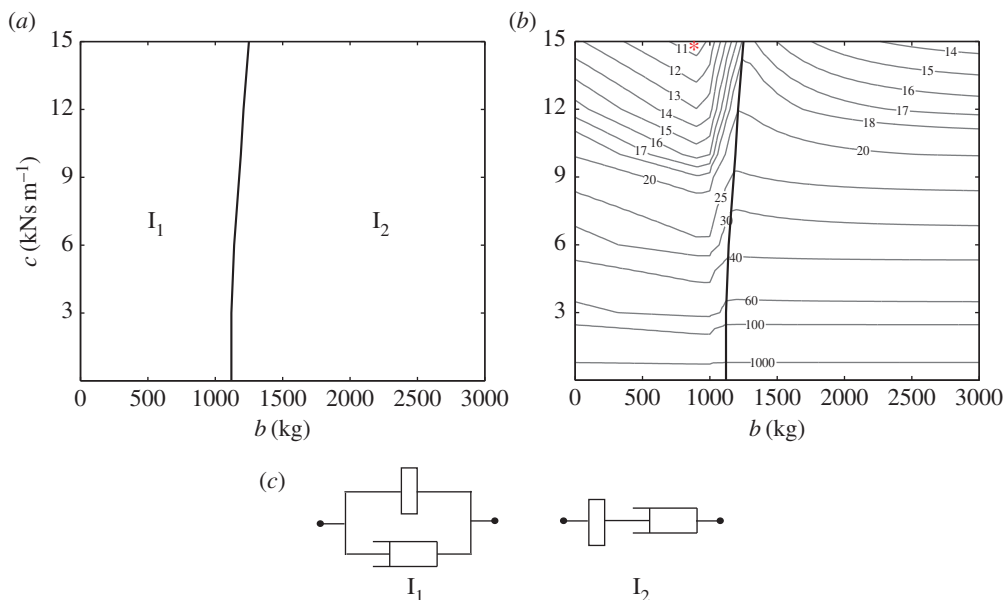


Figure 9. Optimal results for the one damper and one inerter case: (a) optimal structures with different inertia and damping value, (b) the corresponding values of J_∞ and (c) the corresponding optimum structures. (Online version in colour.)

$k = 1500 \text{ kN m}^{-1}$, and consider the displacements of the building storeys relative to that of the base as the performance index. The objective function is defined as [16]

$$J_\infty = \max(\|T_{R \rightarrow Z_i}(j\omega)\|_\infty), \quad i = 1, \dots, n \quad (3.1)$$

where $T_{R \rightarrow Z_i}$ denotes the transfer function from R to Z_i and $\|T_{R \rightarrow Z_i}(j\omega)\|_\infty$ is the standard H_∞ -norm, which represents the maximum magnitude of $T_{R \rightarrow Z_i}$ across all frequencies. We note that a wide variety of cost functions could be studied, but here it is the approach we take to address the optimization problem that is important rather than the detailed cost function dependent results.

The suppression system $Y(s)$ is taken to be a inerter-based vibration device including one inerter and one damper. The range of the inerter is chosen as $b \in [0, 3000 \text{ kg}]$ and that of damping is varying from 10 Ns m^{-1} to 15 kNs m^{-1} . To obtain the optimal structure with a specific value of b and c , we optimize the objective function J_∞ with the functions ((2.1)– (2.4)), making use of patternsearch and fminsearch in Matlab. For example, using the function (2.2) as the transfer function of the suspension device, four parameters need to be optimized; however, at any time three of them will be zero due to the constraints, simplifying the optimization space considerably. Hence, the optimization can provide accurate results including the optimal configuration and the corresponding value of J_∞ . The optimal structures and the corresponding optimal results for the three different predetermined number of springs (zero, one and two) with respect to the value of the inerter and of the damper have been shown in figures 9, 10 and 11, respectively.

The results in figure 9a show that, for the case where the suppression device has a damper and inerter but no springs, the two possible structures, shown in appendix A, table 1, both have regions in the parameter space where they are the optimal layouts. The optimal results in terms of J_∞ (3.1) have also been presented in figure 9b across the inertia–damping parameter space. The minimum value of J_∞ is 10.47 with $b = 897.9 \text{ kg}$ and $c = 15 \text{ kNs m}^{-1}$, which has been shown as the asterisk in figure 9b. It can also be seen that, in the range of $b \in [900, 1200 \text{ kg}]$ and $c \in [9.5, 15 \text{ kNs m}^{-1}]$, the optimal results are very sensitive to the change of the values of b and c . It can also be noted that increasing the inertia does not always obtain a better performance, for

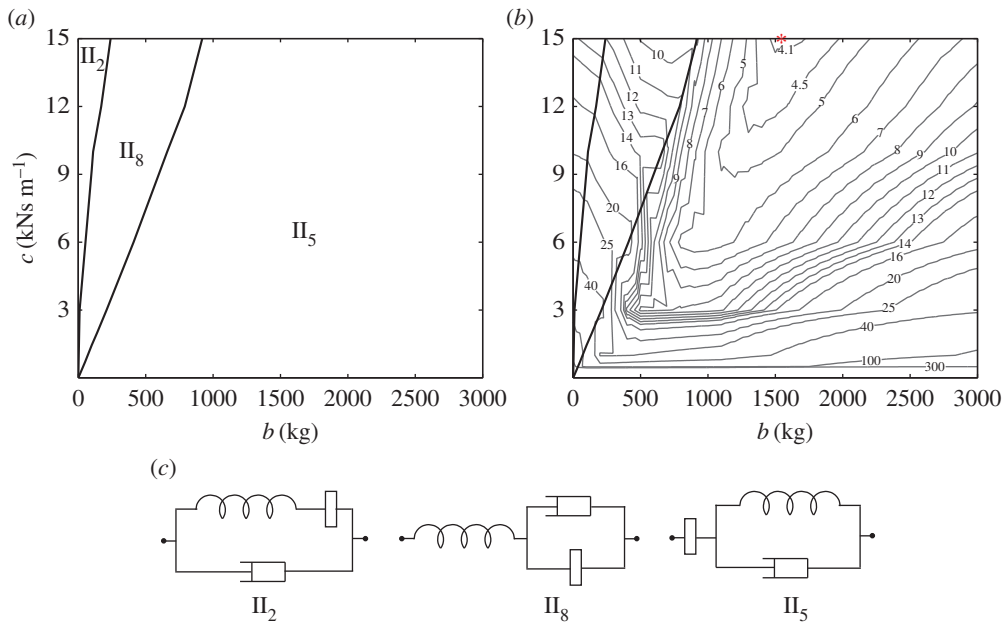


Figure 10. Optimal results for the one-damper, one-inerter and one-spring case: (a) optimal structures with different inertia and damping value, (b) the corresponding values of J_∞ and (c) the corresponding optimum structures. (Online version in colour.)

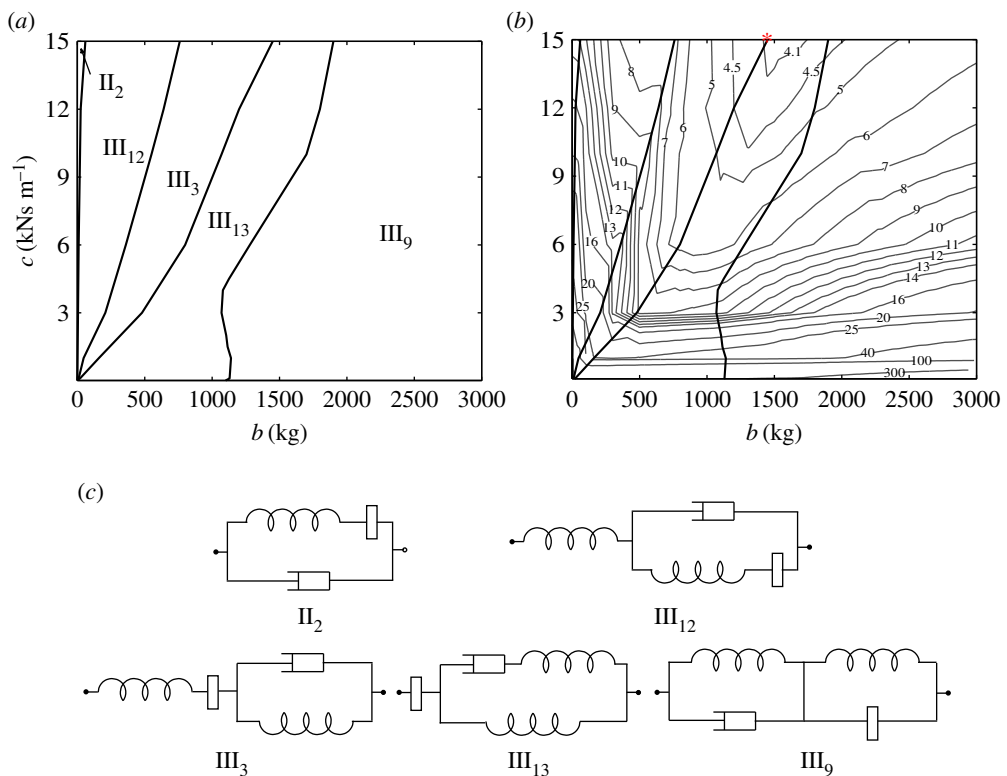


Figure 11. The optimal results for one damper, one-inerter and two-springs case: (a) optimal structures with different inertia and damping value, (b) the corresponding values of J_∞ for up to two-spring case and (c) the corresponding optimum structures. (Online version in colour.)

example, when $c = 13 \text{ kNs m}^{-1}$, $b = 750 \text{ kg}$, the value of J_∞ is about 12; however, if b is changing to 2500 kg , J_∞ increases to approximately 15, almost 25% larger.

For the case where the suppression device includes one spring, out of the eight possible layouts² (see appendix A, table 2) obtained using the generic networks of figure 6, the optimization indicates that three networks are optimal across the b, c region considered (figure 10a). Figure 10c shows these layouts. The corresponding optimal values of the objective function J_∞ (3.1) are shown in figure 10b, where the asterisk represents the minimum value of the objective function with $J_\infty = 4.01$ when $b = 1600.6 \text{ kg}$ and $c = 15 \text{ kNs m}^{-1}$. Comparing with the previous case with no springs, the performance improves about 62%. Considering figure 10b further, we can see that the structure Π_5 is very sensitive to the change of the parameters b and c in some regions. Comparing with the optimal results shown in figure 9b, it can be seen that with the same value of the inertance and the damper, the suppression device with an additional spring can always provide a much better performance. In addition, if we want to get a acceptable value of J_∞ , say $J_\infty = 12$, with no spring structure, I_1 could be chosen with $b = 750 \text{ kg}$, $c = 13 \text{ kNs m}^{-1}$, whereas with a spring, $J_\infty = 12$ can be obtained with a smaller b and c , 500 kg and less than 12 kNs m^{-1} , respectively.

Allowing two springs in the suppression device gives rise to the 18 possible structures (see appendix A, table 3), contained in the two generic networks shown in figure 7-Step 5. Of these, five layouts are optimal over the b - c region considered. These are shown in figure 11c with the corresponding regions given in figure 11a. Figure 11b provides the optimal results of J_∞ with these five structures in the range of the inertance b and the damping value c . The structure Π_2 has one spring which means that in its corresponding optimal region, adding another additional spring in any location is deleterious. The minimum optimal result with the suppression device consisting of up to one damper, one inerter and two springs is $J_\infty = 3.93$ with $b = 1502 \text{ kg}$ and $c = 15 \text{ kNs m}^{-1}$, identified by an asterisk in figure 11b. The minimum value of J_∞ is similar to that of the one spring case, with only about a 1.8% improvement. However, compared with the one-spring case, it can be seen that the range for the values of b and c that can provide the same performance is larger for the two spring case. For example, for the value of J_∞ equal to 5, it can be seen from figure 11b, $b \in [1000, 2450 \text{ kg}]$ and $c \in [8.7, 15 \text{ kNs m}^{-1}]$ and from figure 10b, $b \in [1150, 2250 \text{ kg}]$ and $c \in [9, 15 \text{ kNs m}^{-1}]$. In addition, the performance of the suppression device including two springs can always have a better performance than that of the device including one spring for the same value of b and c . When $b = 200 \text{ kg}$ and $c = 6 \text{ kNs/m}$, for the two-spring case, J_∞ is about 15, and for the one-spring case, J_∞ is approximately 24 with almost 60% larger. It has been shown in figures 9–11 that the structure-immittance method is able to provide detailed optimization results across parameter ranges for structures with specified complexity in terms of the numbers of each component used. In doing this, not only are we able to find the optimal configuration for a certain range of inertance and damping values, but we are also able to identify the regions in the b - c parameter space where each possible layout is optimal.

With this example, the benefits of the proposed structure-immittance approach can also be demonstrated over the structure-based or immittance-based approach. With the structure-based approach, only one device structure is considered during the optimization, such as the TID studied in [12,15] and the TVMD proposed by Ikago [13]. The structure-immittance approach reveals where in the b - c parameter space these are optimal compared with devices of comparable complexity— Π_5 and Π_8 in figure 10a are the TID and TVMD layouts, respectively. In [16], a sophisticated immittance-based approach was applied to designing a vibration suppression device in a building model. The candidate layout consisting of two bilinear transfer functions and an inerter was considered to ensure the vibration suppression device has one and only one inerter. From this, a complicated seven-element structure consisting of one inerter, two springs and four dampers was obtained. Using a simplification process, the IPD (I_1), TID (Π_5) and TTID (Π_9) were obtained as the approximate optimal structures for the different inerter's sizes.

²Note that within these layouts, element sizes can go to zero or infinity, allowing the layouts of fewer numbers of components to also be identified; they represent special cases of this category.

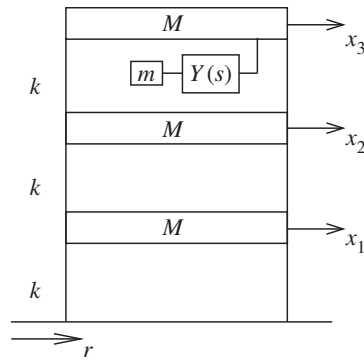


Figure 12. The three-storey building model with an attached mass and a two-terminal vibration suspension device.

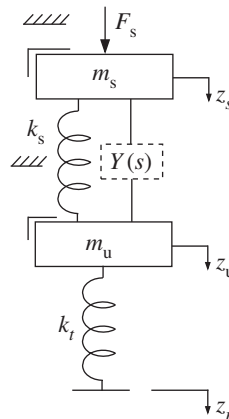


Figure 13. Quarter-car vehicle model with predetermined static stiffness.

However, some optimal configurations of comparable complexity, such as Π_2 , III_{12} (shown in figure 11), obtained in this paper using the structure–immittance approach, cannot be realized by the fixed-sized-inerter layout used in [16].

Following on from §2d and motivated by the vast literature on TMDs, the case where an additional mass is used as a part of the suppression system is now considered. As described in §2d, to fit with the structure–immittance approach, the mass is treated as an optimizable part of the structure. This is illustrated using the example building model shown in figure 12, with the floor mass M and the inter-storey elasticity k taken to be those used in the previous example. The small mass m is assumed to be 150 kg, 5% of the structural mass. By optimizing the objective function (3.1) with a traditional TMD, that is $Y(s)$ represents a parallel connection of spring k_d and damper c_d , we obtain $J_\infty = 24.19$ with $k_d = 47.4 \text{ kN m}^{-1}$ and $c_d = 3.73 \text{ kNs m}^{-1}$. Now, we use the structure–immittance approach with $Y(s)$ including one damper, one inerter and one spring, which corresponds to the force–velocity transfer functions given in (2.2) and (2.3). This reveals that the network Π_2 shown in table 2, appendix A, provides the best performance with $J_\infty = 18.59$ when $k_d = 50.98 \text{ kN m}^{-1}$, $c_d = 6.998 \text{ kNs m}^{-1}$ and $b = 178 \text{ kg}$. It can be seen that this inerter-based vibration suppression device provides a 23.1% improvement compared with the conventional TMD.

(b) Vehicle suspension for quarter-car model

The quarter-car model presented in figure 13 is the simplest model to consider for vehicle suspension design. It consists of the sprung mass m_s , the unsprung mass m_u and a tyre with

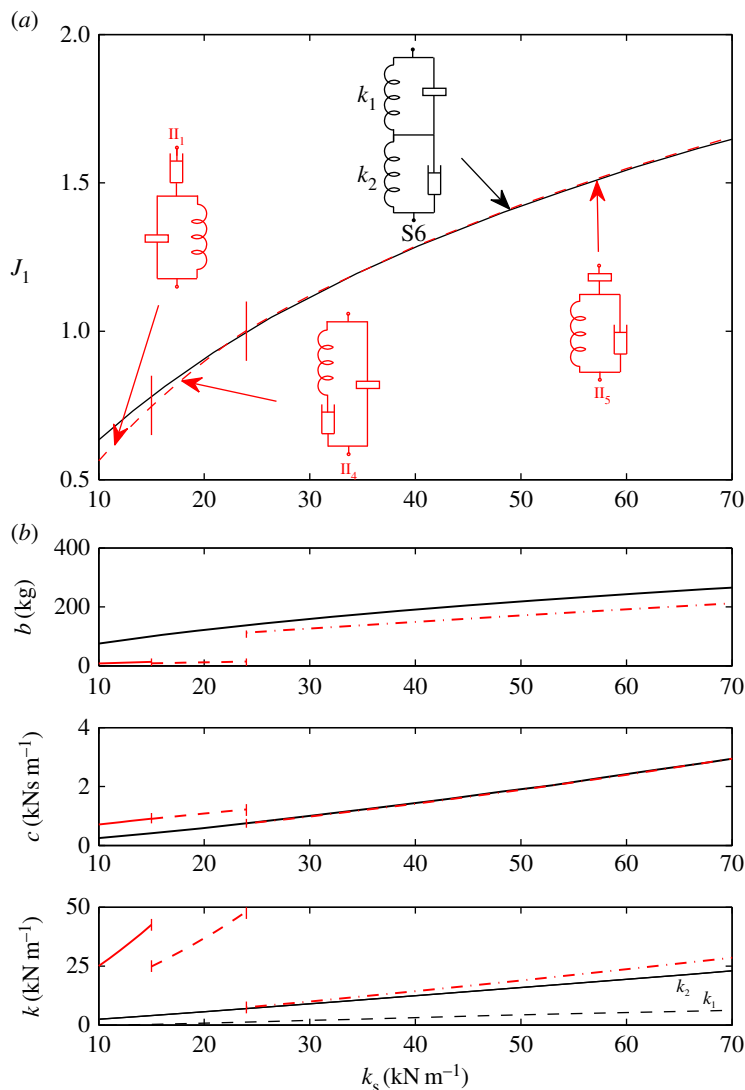


Figure 14. The optimization results: (a) the values of J_1 with layout S6 (black solid line), structural immittances (red dashed line), (b) the corresponding optimal parameter values for S6 (black dashed line for k_1 and black solid line for k_2), Π_1 (red solid), Π_4 (red dashed) and Π_5 (red dash dotted line). (Online version in colour.)

spring stiffness k_t . The suspension strut provides an equal and opposite force on the sprung and unsprung masses and is assumed to be a passive transfer function $Y_t(s) = k_s/s + Y(s)$, where the device admittance is supplemented with a spring of stiffness k_s to ensure the suspension system has sufficient static stiffness. The parameters of the quarter-car model are taken to be $m_s = 250$ kg, $m_u = 35$ kg, $k_t = 150$ kN m^{-1} , as used in [8,27].

For brevity, only the ride comfort performance measure J_1 defined in [8] is investigated, and

$$J_1 = 2\pi(V\kappa)^{1/2} \|sT_{\hat{z}_r \rightarrow \hat{z}_s}\|_2$$

where V is the forward velocity of the vehicle, κ is a road roughness parameter and $T_{\hat{z}_r \rightarrow \hat{z}_s}$ denotes the transfer function from \hat{z}_r to \hat{z}_s . As used in [8,27], we take $V = 25$ m s^{-1} and $\kappa = 5 \times 10^{-7}$ $\text{m}^3 \text{ cycle}^{-1}$.

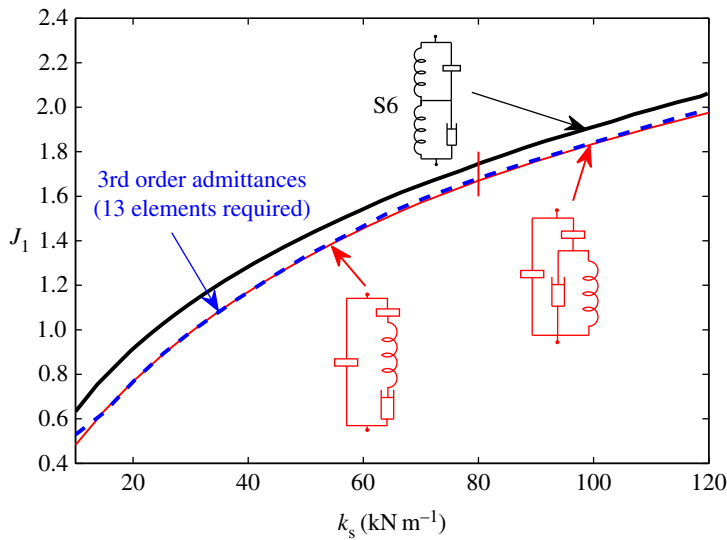


Figure 15. The optimization of J_1 on: layout S6 (black bold), third-order admittances (blue dashed line) and structure-immittance functions for one-damper, one-spring and two-inerters case (red solid line). (Online version in colour.)

In [8], for the same quarter-car model, the structure-based approach with eight candidate layouts was applied to investigate the suspension performance. Later, Papageorgiou & Smith [27] showed a performance improvement using the immittance-based approach with a third-order immittance function. Here, we will minimize J_1 using the proposed structure-immittance approach and compare the results with those obtained in [8,27].

Considering a suspension device with one damper, one inerter and at most two springs (in addition to the spring in parallel to the device), the structural immittances (2.4), identified in §2, have been applied. In line with the study in [8,27], the optimization is run for fixed values of static stiffness k_s in the range 10–120 kN m^{-1} . The optimal results are shown in figure 14a as a dashed red line. The short vertical lines show the transmission point between the optimal layouts with respect to the value of static stiffness, with the three layouts Π_1 , Π_3 and Π_5 shown in the figure. Now consider the results reported by Smith & Wang [8] using the structure-based approach; their S6 device layout provided the best performance, and is shown as the black solid line in figure 14a. Note that for $0 < k_s \leq 120 \text{ kN m}^{-1}$ (120 kN m^{-1} is the maximum stiffness considered in [8]), layout S6 and the optimal structure Π_5 have similar performance. It can be seen that compared with results reported in [8] using layout S6, the obtained structures Π_1 and Π_4 can have better performance and Π_5 results in similar results over the range they are optimal for. An added benefit is that as these three structures only have one spring, they are simpler than layout S6. The corresponding parameter values are shown in figure 14b.

Now consider a suspension device with one damper, one spring and two inerters. Based on the general formulation flowchart shown in figure 5, the structural immittances can be obtained. Using these functions as the suspension transfer functions, $Y(s)$, for optimization, results are shown as a red line in figure 15. For the sake of comparison, the values of J_1 obtained with a third order immittance function [27] have also been plotted (blue dashed line), along with optimal results for the fixed structure S6 [8] (black line). The two optimum configurations including two inerters are shown in figure 15 for the range of $k_s \in [10 \text{ kN m}^{-1}, 80 \text{ kN m}^{-1}]$ and $k_s \in (80 \text{ kN m}^{-1}, 120 \text{ kN m}^{-1}]$, respectively. Figure 15 suggests that comparing with the fixed structure layout S6, the structure-immittance approach with two inerters provides much better performance, around 22% improvement. Also they achieve similar optimum results to those of a third-order admittance function in the greatest range of k_s , from 13 to 120 kN m^{-1} . When k_s is smaller than 13 kN m^{-1} ,

the value of J_1 obtained with the structure–immittance approach is slightly better than that of the third-order function. Furthermore, based on Bott–Duffin procedure, to realize the third-order function, 13 elements consisting of four inerters, five dampers and four springs are required, which is much more complicated than the structures obtained with the structural immittances.

4. Conclusion

A new approach for the optimization of passive vibration control device, namely the structure–immittance approach, has been proposed. It allows a full set of networks with a predetermined number of each element type to be analysed systematically with structural immittances, which are obtained using the proposed formulation process. In addition to being able to manage the complexity of the layout in terms of the number of elements, the value of each element can be restricted within the optimization if desirable, representing a significant advantage over the immittance-based approach. By using these structural immittances as the force–velocity transfer functions to represent the dynamics of the vibration suppression device for a building and a quarter-car model, the advantages of the proposed approach have been shown. When considering the building applications, the number of inerter and dampers was restricted to be one and for different number of springs, optimal configurations were obtained over a range of inertia and damping values. Furthermore, the approach indicates the sensitivity of the device. These can provide guidance for selecting the appropriate suspension device considering the element numbers, the element values and the robustness. In the example of automotive control, the new structure–immittance approach was used to match performance levels reported in the literature using the structure-based approach but with simpler layouts. In addition, with the structural immittances for one damper, one spring and two inerters, two simple configurations were obtained, which can provide similar and even better performance, compared with those obtained using the immittance-based approach with a third-order admittance function which in general requires 13 elements to be realized.

Data accessibility. The datasets supporting this article are already available in the article.
Authors' contributions. S.Y.Z. led the development of the method with assistance from J.Z.J. and S.A.N. All three contributed to the preparation of the manuscript.
Competing interests. We declare we have have no competing interests.
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Appendix A. Networks covered by generic networks shown in figures 2, 6-Step 5 and figure 7-Step 5

Table 1. A full set of networks with one damper, one inerter and no spring covered by the generic network shown in figure 2.

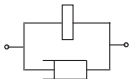
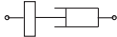
I_1		I_2	
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Table 2. A full set of networks with one damper, one inerter and one spring covered by generic networks shown in figure 6-Step 5.

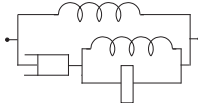
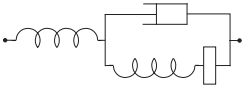
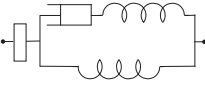
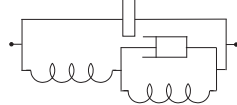

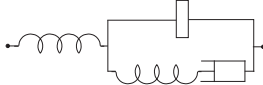

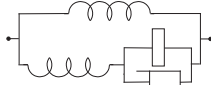
left-hand network of figure 6-step 5	right-hand network of figure 6-step5
Π_1 	Π_2
Π_3 	Π_4
Π_5 	Π_6
Π_7 	Π_8

Table 3. A full set of networks with one damper, one inerter and two springs covered by generic networks shown in figure 7-Step 5.

left-hand network of figure 7-step 5	right-hand network of figure 7-step 5
III_1 	III_2
III_3 	III_4
III_5 	III_6
III_7 	III_8
III_9 	III_{10}

(Continued.)

Table 3. (Continued.)

III ₁₁		III ₁₂	
III ₁₃		III ₁₄	
III ₁₅		III ₁₆	
III ₁₇		III ₁₈	

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